Math 104 | **Midterm Exam 1 v. B** | **Name:** ____________
October 4, 2019 | **(Total Points: *70*)** | **SID:** ____________

Information

1. **This is a closed-book test; no notes nor books are permitted.**
2. **This is a closed-phone test; no use of electronics is permitted.**
3. **Read each question carefully and answer each question completely.**
4. **Please be precise when writing your answers, and use complete sentences where appropriate (especially in proofs). You may use arithmetic manipulations freely, and you may use any result from the lectures or homework so long as:**
   - **you mention explicitly the result you are using;**
   - **the result you are using is not the same one you are being asked to prove; and**
   - **the result you are using was proved without using the result you are being asked to prove.**
5. **I will give partial credit to partial arguments; however, mere lists of facts (even true facts) that are going nowhere will receive very little or no credit. Arguments based on intuitive concepts or pictures may receive partial credit, but for full credit, a complete formal proof is required.**
6. **Write your name at the top of every page.**

Question	Points
1	15
2	20
3	15
4	20
Total:	70

 Name: ____________________

(5 points) 1. (a) Define what an *order* on a set S is.

(10 points) (b) Suppose that S is an ordered set with order $\preceq$, $E \subseteq S$, and there is $e \in E$ which is an upper bound for E. Show that $e = \sup E$.

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(5 points) 2. (a) Define the *closure* of a set.

(10 points) (b) Let $E \subseteq \mathbb{R}$ be bounded above. Prove that $\sup E \in \overline{E}$.

(5 points) (c) Must $\sup E$ be a limit point of E?

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(5 points) 3. (a) Define what is meant by the ball $B_r(x)$ in a metric space (M, d).

(10 points) (b) Prove that if (M, d) is a metric space and $x \in M$, then $B_r(x)$ is open.

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(5 points) 4. (a) Define what it means for a sequence to be *convergent*.

(10 points) (b) Suppose that $(a_n)_n$ is a sequence in a metric space (M, d). Show that if $(a_n)_n$ converges, then $A = \{a_n \mid n \in \mathbb{N}\}$ has at most one limit point in M.

(5 points) (c) Is the converse true? That is, if $(a_n)_n$ is a sequence in (M, d) so that $A = \{a_n \mid n \in \mathbb{N}\}$ has at most one limit point, must $(a_n)_n$ converge?